The Konigsberg Bridge Problem is a classic problem, based on the topography of the city of Konigsberg, formerly in Germany but now known as Kaliningrad and part of Russia. The river Pregel divides the city into two islands and two banks as shown in figure 1. The city had seven bridges connecting the mainland and the islands (represented by thick lines in the figure). [1][2][3][6].

![Konigsberg Bridge Problem Diagram](image)

The problem was to begin at any point, cross all seven bridges exactly once and return to the starting point. No one succeeded in doing this. A Swiss mathematician Leonhard Euler formulated this problem by abstracting the scenario. He formulated the problem as finding a sequence of letters A, B, C, D (that represent the land areas) such that the pairs (A,B) and (A,C) appear twice (thus representing the two bridges between A and B, and A and C) and the pairs (A,D), (B,D), (C,D) appear only once (these pairs would represent the bridges between A and D, B and D, C and D). Euler used a counting argument to prove that no such sequence exists thus proving that there the Konigsberg Bridge Problem has no solution. This result was presented by Euler in a paper, “The Solution of Problem Relating to the Geometry of Position” at the Academy of Sciences of St. Petersburg in 1735. This paper, in addition to proving the non-existence of solution to the Konigsberg Bridge Problem, gave some insights into general arrangements of bridges and land areas [11].

Euler summarized his main conclusions as follows:
1) If there is any land area that is connected by an odd number of bridges, then a cyclic journey that crosses each bridge exactly once is impossible.
2) If the number of bridges is odd for exactly two land areas, then there is a journey that crosses each bridge exactly once is possible, if it starts at one of these areas and ends in the other.
3) If there are no land areas that are connected by odd number of bridges, the journey can start and end at any land area.[11]

Euler gave heuristic reasons for the correctness of the conclusions, but did not propose formal proofs. [4][7][11]
The paper presented by Euler on the Konigsberg bridge problem can be considered to mark the birth of graph theory, in general. Later, a diagrammatic representation evolved which involved nodes or vertices and the connecting lines are called edges. Using this representation, Konigsberg problem is modeled as shown in figure 2. The islands and banks are represented by circles called nodes and the bridges are represented by connecting lines called edges. The number of edges that are incident on a node is called the degree of the node [5]. In the Konigsberg bridge problem, the number of bridges connecting a land area would be the degree of the node representing the land area.

While studying the Konigsberg bridge problem, Euler also made the observation that the number of bridges at every land area would add up to twice the number of bridges. This result came to be known as the hand-shaking lemma in graph theory which states that the sum of node-degrees in a graph is equal to twice the number of edges. This result is the first formulation of a frequently used result in graph theory that states that the sum of node-degrees in a graph is always even [7],[9],[11],[12]. The results from the solution of the Konigsberg problem have been extended to various concepts in graph theory. In graph theory a path that starts and ends at the same node and traverses every edge exactly one is called an Eulerian circuit. The result obtained in the Konigsberg bridge problem has been generalized as the Euler’s theorem which states that a graph has an Eulerian circuit if and only if there are no nodes of odd degree. Since the graph corresponding to Königsberg has four nodes of odd degree, it cannot have an Eulerian circuit. Subsequently the concept of Eulerian paths was introduced which deals with paths that traverse every edge exactly once. It was proved that such a path exists in a graph if and only if the number of nodes of odd degree is 2. [7],[9],[11][12].

An Eulerian cycle is the minimum length path that traverses every edge exactly once. If it exists, it solves the snowplow problem which finds the minimum distance path in a road network. However, it is unlikely that any real road network would happen to satisfy the degree conditions that make it Eulerian. In that case, the problem moves to the realm of “Chinese Postman problem” [10],[11],[12]. The impact that the Konigsberg problem made is powerful. It paved the way for the creation of a new modeling theory called graph theory. The applications of graph theory are numerous. It has wide applications in science and engineering. The concept of “geometry of position” in the first paper Euler presented, inspired the formulation of a new field Topology which is a significantly important branch of Mathematics.

Relationships between eulerian graphs and other graph properties have been studied