Shapes of Bushy Join Tree

Given a specified join ordering in a multirelation queries of n tables, the number of different bushy join tree is given by the following recurrence relation (i.e., recursive function), with \( S(n) \) defined as follows: \( S(1) = 1 \).

\[
S(n) = \sum_{i=1}^{n-1} S(i) \cdot S(n - i)
\]

More detailed explanation can be found in section 19.5.3 (page 721-723) of the textbook.

Moreover, \( S(n) = \text{Catalan}(n - 1) \). “Using zero-based numbering, \( \text{Catalan}(n) \) is given directly in terms of binomial coefficients by

\[
\text{Catalan}(n) = \frac{1}{n + 1} \binom{2n}{n} = \prod_{k=2}^{n} \frac{n + k}{k}
\]

The first Catalan numbers for \( n = 0, 1, 2, 3, 4, 5, \ldots \) are 1, 1, 2, 5, 14, 42, ...”

“Successive applications of a binary operator can be represented in terms of a full binary tree. (A rooted binary tree is full if every vertex has either two children or no children.) It follows that \( \text{Catalan}(n) \) is the number of full binary trees with \( n + 1 \) leaves:”