Kinetic prisms:
incorporating acceleration limits into space-time prisms
ABSTRACT
Presently, data concerning moving objects abound. These data mainly consist of time-stamped geographical locations, which are collected by location aware devices, such as Global Positioning System receivers. Space–time prisms are used to model the spatio-temporal space of potential movement in between measured locations (called anchors). They rely on the knowledge of the maximal speed of travel of an object and they capture all space–time paths that respect this speed limit. However, the classic space–time path and prism model is not physically realistic, in the sense that it contains spatio-temporal paths of moving objects can alter their direction and speed instantaneously. Since this is physically impossible, the classical model is not acceptable in applications where mechanics and kinetics are vital. We propose a more realistic version of space–time prisms, in which not only speed but also acceleration is bounded. This additional bound results in a physically realistic model, which we refer to as kinetic prisms. Furthermore, we study how imposing constraints on the speed and heading at anchor points affects the geometry of kinetic prisms. In this paper, we give analytical descriptions of kinetic prisms and algorithms for their construction for movement in one- and two-dimensional space.
Motivation: Societal Importance

Figure 1: News clippings on missing ship trajectories

BBC News, 2014

BBC Future, 2018

The New York Times, 2019

management rules. As a result, illegal fishing has become a multi-billion-dollar industry, worth up to $23bn each year. Because of overfishing – both legal and illegal – one third of fisheries assessed in a study by the UN Food and Agriculture Organisation were overfished and over half were fully fished. This threatens jobs and food security for millions of people, all around the world.
Motivation: Societal Importance

Figure 2: A long trajectory gap around Galapagos Reserve
Natural Geographic, 2018
Problem statement

- **Input:**
  - Locations of a moving object in space-time with gaps
  - Bounds (e.g: maximum speed, maximum acceleration)
  - Time range

- **Output:**
  - All possible locations

- **Goal:**
  - Model the potential movement

- **Constraint:**
  - Geometric dimensions (up to 2-dimensions)
  - Modeling change in speed or acceleration

Figure 3: Problem statement
Key concepts

Trajectory: Path of movement

Classical space-time prism: all possible paths between anchor points within the time range, given the object’s maximum speed

Figure 4: Classical space-time prism
Limitations of Related Work

- The **classical space-time model** is physically unrealistic
  - Does not model acceleration bound
  - Assumption: Simultaneous speed and direction change
  - This will require infinite acceleration
- Overestimate the space that a moving object can reach

Figure 5: Classical space-time prism
Major Contribution

- **Kinetic space-time model**: incorporated acceleration bound
- Investigated the effects of adding conditions on measured locations (called anchors)
  - Anchor speed and heading (the directional components of the speed)

![Figure 6: An illustration of anchor speed and heading](image-url)
Key concepts

Kinetic space-time path: trajectory with speed and **acceleration bounds**

Kinetic space-time prism: subset of the space–time between two consecutive anchor points that contain all points that belong to some kinetic path

![Figure 7: Three case studies on Kinetic space-time prism](image)
Validation methodology

1. Lemmas to support geographic description in use cases

Example: Let the anchors be p and q, maximal speed $v_{\text{max}}$, maximal acceleration $a_{\text{max}}$

the middle point $m = ((x_p + x_q)/2, (t_p + t_q)/2)$ of its anchor points

Then, we have Reflection at middle point m

$$(\mathcal{KP}(p, q, v_{\text{max}}, a_{\text{max}})) = \mathcal{KP}(p, q, v_{\text{max}}, a_{\text{max}})$$

This lemma implies that it is enough to describe the left border of the kinetic prism, since its right border can be obtained from it.
Validation methodology

2. Performed visualization of 3 examples using Mathematica

Figure 8: Three types of kinetic prism (with no anchor conditions) in two dimensions.
Validation methodology

Strength

- Geometric description gives more access to readers with less mathematical backgrounds, as they need not spend time understanding notations

Weakness

- Lacks validation using experiments
Assumptions

Number of dimensions

- Considers kinetic prisms for 1 and 2 dimensions
- Remove assumption: add an additional dimension $z$ to the anchor points

Point model of anchors

- Consider anchors as a shape that models uncertainty (error and object’s shape)
Limitation of This Work

- Has not explored kinetic prism in road transportation network
  - The current model assumes constant maximum velocity.
  - Road networks have different speed limits.
  - Modeling different speed limits might result in different prisms
Things to be preserved

- Presentation of the idea:
  - Building up from the simplest, most unrealistic case.
  - Geometric description
  - Visualizations

- States situations where the classical model might be preferred due to less complexity
  - In transportation and migration, mechanics and kinetics might be less relevant.

- Clearly stating major assumptions and limitations
Revisions to be made

- Address the assumptions made on geographic dimensions (1 and 2)
- Put societal importance in the motivation
- Add inequality equations for the prism model involving the acceleration and velocity bounds
- Improve validation by performing experiments
Exercises on key concepts

1. What is the main difference between the classical space-time prism and the kinetic space-time prism?
   a. Speed bound was only considered in classical space-time prism
   b. Speed bound was only considered in kinetic space-time prism
   c. Acceleration bound was only considered in classical space-time prism
   d. Acceleration bound was only considered in kinetic space-time prism
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2. Identify the kinetic and classical space-time prisms in the below figure: